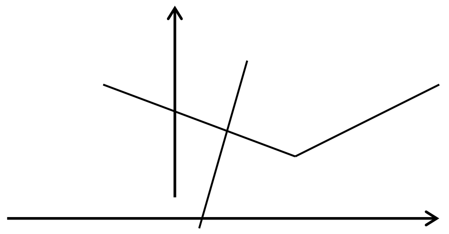
MARK SCHEME

1.



M1 A1

$1 - x = 6x - 1$ M1

$x = {}^2/_7$ A1

$x < {}^2/_7$ A1

2.(a) $I = e^{-\int \frac{1}{t} dt} = e^{-\ln t} = \frac{1}{t}$ M1, A1, A1

$\frac{d}{dt}\left(\frac{v}{t}\right) = \frac{1}{t}$ $\quad \frac{v}{t} = \ln t + c$ M1 A1 $\quad v = t(\ln t + c)$ A1

(b) $v = 3$, $t = 2$, $c = {}^3/_2 - \ln 2$ M1, A1 $\quad t=4$, ${}^v/_4 = \ln 4 + {}^3/_2 - \ln 2 = 8.77$ M1, A1

3.(a) $y' = ½x^2e^x + xe^x$ B1 $\quad y'' = ½x^2e^x + 2xe^x + e^x$ B1

$y'' - 2y' + y = ½x^2e^x + 2xe^x + e^x - x^2e^x - 2xe^x + ½x^2e^x$ M1

$= e^x$ A1

(b) Aux. eqn $m^2 - 2m + 1 = 0$ $\quad m = 1$ M1, A1

C.F. $\quad e^x(A + Bx)$ A1

Gen. soln. $\quad y = e^x(A + Bx) + ½x^2e^x$ A1ft

$x = 0$, $y = 1$ $\quad A = 1$ B1

$y' = e^x(A + Bx) + Be^x + xe^x + ½x^2e^x$ M1

$y' = 2$, $x = 0$ $\quad 2 = A + B$, $\quad B = 1$ M1, A1

$y = e^x(1 + x + ½x^2)$ A1 ft

4.(a) Circle B1, Diameter 3a B1

Cardiod cusp at O B1 Symmetry and 2a B1

(b) $3a\cos\theta = a(1 + \cos\theta)$, $\cos\theta = \frac{1}{2}$ M1

$\theta = \pm {}^{\pi}/_{3}$, $r = {}^{3a}/_{2}$ A1, A1

(c) $A_1 = \frac{1}{2}\int a^2(1+\cos\theta)^2\,d\theta$ M1

$= \frac{1}{2}a^2\int\left(1 + 2\cos\theta + \frac{1}{2}(1+\cos 2\theta)\right)d\theta$ M1, A1

$= \frac{1}{2}a^2\left[\frac{3\theta}{2} + 2\sin\theta + \frac{1}{4}\sin 2\theta\right]$ A1, A1

Evaluate A_1 using 0 and ${}^{\pi}/_{3}$ M1

$A_1 = \frac{\pi a^2}{4} + \frac{9\sqrt{3}a^2}{16}$ A1

(d) Area required = ${}^{9}/_{4}\pi a^2 - 2A_1 - 2\times$given M1 ${}^{9}/_{4}\pi a^2$ B1

$= \frac{9\pi a^2}{4} - \frac{\pi a^2}{2} - \frac{9\sqrt{3}a^2}{8} - \frac{3\pi a^2}{4} + \frac{9\sqrt{3}a^2}{8}$ M1

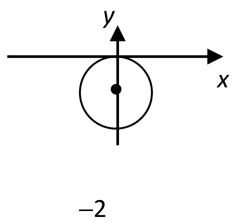
$= \pi a^2$ A1

Question Number	Scheme	Marks
5.	$(x > 0)$ $2x^2 - 5x > 3$ or $2x^2 - 5x = 3$	M1
	$(2x + 1)(x - 3)$, critical values $-\frac{1}{2}$ and 3	A1, A1
	$x > 3$	A1 ft
	$x < 0$ $2x^2 - 5x < 3$	M1
	Using critical value 0: $-\frac{1}{2} < x < 0$	M1, A1 ft
Alt.	$2x - 5 - \frac{3}{x} < 0$ or $(2x - 5)x^2 > 3x$	M1

		$\dfrac{(2x+1)(x-3)}{x} > 0$ or $x(2x+1)(x-3) > 0$	M1, A1
		Critical values $-½$ and 3, $x > 3$	A1, A1 ft
		Using critical value 0, $-½ < x < 0$	M1, A1 ft
			(7 marks)
6.	(*a*)	$\dfrac{dy}{dx} + y\left(\dfrac{\sin x}{\cos x}\right) = \cos^2 x$	M1
		Int. factor $e^{\int \tan x\,dx} = e^{-\ln(\cos x)} = \sec x$	M1, A1
		Integrate: $y\sec x = \int \cos x\,dx$	M1 , A1
		$y\sec x = \sin x + C$	A1
		$(y = \sin x\cos x + C\cos x)$	**(6)**
	(*b*)	When $y = 0$, $\cos x(\sin x + C) = 0$, $\cos x = 0$	M1
		2 solutions for this $(x = \pi/2, 3\pi/2)$	A1 **(2)**
	(*c*)	$y = 0$ at $x = 0$: $C = 0$: $y = \sin x\cos x$	M1
		$(y = ½\sin 2x)$ Shape	A1
		Scales	A1 **(3)**
			(11 marks)
7.	(*a*)	$2m^2 + 7m + 3 = 0$ $(2m+1)(m+3) = 0$	
		$m = -½, -3$	
		C.F. is $y = Ae^{-½t} + Be^{-3t}$	M1, A1
		P.I. $y = at^2 + bt + c$	B1
		$y' = 2at + b$, $y'' = 2a$	
		$2(2a) + 7(2at + b) + 3(at^2 + bt + c) \equiv 3t^2 + 11t$	M1

		$3a = 3$, $a = 1$ $\quad 14 + 3b = 11$, $b = -1$	A1	
		$4 - 7 + 3c = 0$, $c = 1$	M1, A1	
		General solution: $y = Ae^{-½t} + Be^{-3t} + (t^2 - t + 1)$	A1 ft	**(8)**
	(*b*)	$y' = -½Ae^{-½t} - 3Be^{-3t} + (2t - 1)$	M1	
		$t = 0$, $y' = 1$: $1 = -1 - ½A - 3B$		
		$t = 0$, $y = 1$: $1 = 1 + A + B$ one of these	M1, A1	
		Solve: $A + B = 0$, $A + 6B = -4$		
		$A = {}^4/_5,\ B = -{}^4/_5$	M1	
		$y = (t^2 - t + 1) + \frac{4}{5}(e^{-½t} - e^{-3t})$	A1	**(5)**
	(*c*)	$t = 1$: $y = \frac{4}{5}(e^{-½} - e^{-3}) + 1$ (= 1.445…)	B1	**(1)**
				(14 marks)

8.	(*a*)	$y = r\sin\theta = a(3\sin\theta + \sqrt{5}\sin\theta\cos\theta)$		
		$\frac{dy}{d\theta} = a(3\cos\theta + \sqrt{5}\cos 2\theta)$	M1, A1	
		$2\sqrt{5}\cos^2\theta + 3\cos\theta - \sqrt{5} = 0$		
		$\cos\theta = \frac{-3 \pm \sqrt{9 + 40}}{4\sqrt{5}}$, $\cos\theta = \frac{1}{\sqrt{5}}$	M1, A1	
		$\theta = \pm 1.107…$	A1 ft	
		$r = 4a$	A1 ft	**(6)**
	(*b*)	$2r\sin\theta = 20$	M1	
		$8a\sin\theta = 20$, $a = \frac{20}{8\sin\theta}$ = 2.795…	M1, A1	**(3)**

(*c*)	$(3+\sqrt{5}\cos\theta)^2 = 9+6\sqrt{5}\cos\theta+5\cos^2\theta$	B1
	Integrate: $9\theta+6\sqrt{5}\sin\theta+5\left(\frac{\sin 2\theta}{4}+\frac{\theta}{2}\right)$	M1, A1
	Limits used: $[\ldots]_0^{2\pi} = 18\pi+5\pi$ (or upper limit: $9\pi+\frac{5\pi}{2}$)	A1
	$½\int_0^{2\pi} r^2 d\theta = a^2(23\pi)$ ≈ 282 m^2	M1, A1 **(6)**
		(15 marks)
9. (*a*)(i)	$\lvert x+(y-2)\mathrm{i}\rvert = 2\lvert x+(y+\mathrm{i})\rvert$	M1
	$\therefore x^2+(y-2)^2 = 4(x^2+(y+1)^2)$	
(ii)	so $3x^2+3y^2+12y=0$ any correct from; 3 terms; isw	A1 **(2)**
	 Sketch circle	B1
	Centre (0,–2)	B1
	r = 2 or touches axis	B1 **(3)**
(*b*)	$w = 3(z-7+11\mathrm{i})$	B1
	$= 3z-21+33\mathrm{i}$	B1 **(2)**
		(7 marks)
10. (*a*)	$y\frac{\mathrm{d}^3y}{\mathrm{d}x^3}+\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}; +2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}; +\frac{\mathrm{d}y}{\mathrm{d}x}=0$ marks can be awarded in (b)	M1 A1; B1;B1
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = \frac{-3\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-\frac{\mathrm{d}y}{\mathrm{d}x}}{y}$ or sensible correct alternative	B1 **(5)**
(*b*)	When $x = 0$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-2$, and $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=5$	M1A1, A1 ft
	$\therefore$ $y = 1+x-x^2+\frac{5}{6}x^3\ldots$	M1, A1 ft **(5)**

(*c*)	Could use for $x = 0.2$ but not for $x = 50$ as	B1	
	approximation is best at values close to $x = 0$	B1	**(2)**
			(12 marks)

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